PROBLEM 2

We know that equations of the form $r = \cos(k\theta)$, $r = \sin(k\theta)$

represent flowers with k petals if k is odd,
$2k$ petals if k is even.

There is no need to draw a picture, we only need to find 2 consecutive angles θ for which $r = 0$ (this is when the curve traces one petal).

1) $r = \cos(3\theta)$ $\rightarrow$ 3 petals

$$\cos(3\theta) = 0 \rightarrow 3\theta = \pi/2 + m\pi \rightarrow \theta = \pi/6 + \frac{m\pi}{3}$$

$$\rightarrow \theta = \pi/6,\ \pi/6 + \pi/3,\ \pi/6 + \frac{2}{3}\pi,\ \dots$$

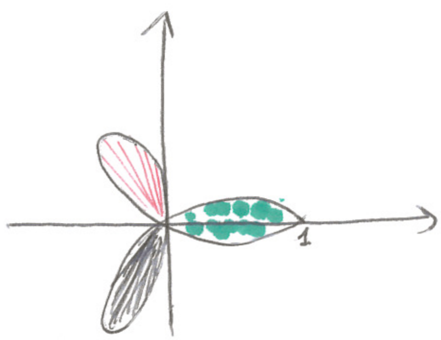
So one petal is (for example) $\pi/6 \le \theta \le \frac{\pi}{6} + \pi/3 = \pi/2$

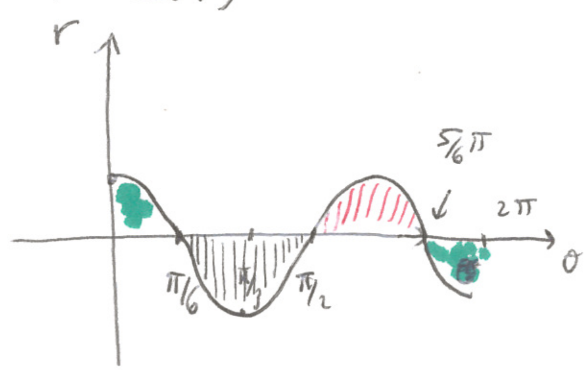
$$\text{Area of the petal} = \int_{\pi/6}^{\pi/2} \frac{1}{2} r^2 \, d\theta = \int_{\pi/6}^{\pi/2} \frac{1}{2} \cos^2(3\theta) \, d\theta$$

$$= \int_{\pi/6}^{\pi/2} \frac{1}{2} \frac{1 + \cos(6\theta)}{2} \, d\theta = \int_{\pi/6}^{\pi/2} \frac{1}{4} \, d\theta + \int_{\pi/6}^{\pi/2} \frac{\cos(6\theta)}{4} \, d\theta$$

$$= \frac{1}{4}\left(\frac{\pi}{2} - \pi/6\right) + \cancel{\left.\frac{\sin(6\theta)}{24}\right]_{\pi/6}^{\pi/2}} = \pi/12$$

if you want the picture (since they are cool)



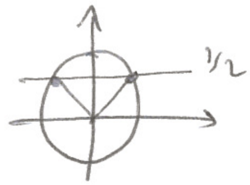


2) Again, we can solve the problem without drawing a graph, although it is always usefue if you can sketch it.

points of intersection : $6\sin\theta = 2+2\sin\theta$

$\rightarrow \sin\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6} + 2n\pi$

$\pi - \frac{\pi}{6} + 2n\pi$



so $\theta = \frac{\pi}{6}, \frac{5}{6}\pi$ in $[0, 2\pi]$

Now we can determine the outer curve by plugging a test point in the interval (for ex $\pi/2$)

$6\sin\pi/2 = 6$

$2+2\sin\pi/2 = 4$

$\rightarrow \text{Area} = \int_{\pi/6}^{5/6\pi} \frac{1}{2}(6\sin\theta)^2 - \frac{1}{2}(2+2\sin\theta)^2 \, d\theta$

$5/6\pi$

$$= \frac{1}{2}\int_{\pi/6}^{5/6\pi}\left(36\sin^2\theta - 4 - 4\sin^2\theta - 8\sin\theta\right)d\theta$$

$$= \int_{\pi/6}^{5/6\pi}\left(16\sin^2\theta - 2 - 4\sin\theta\right)d\theta$$

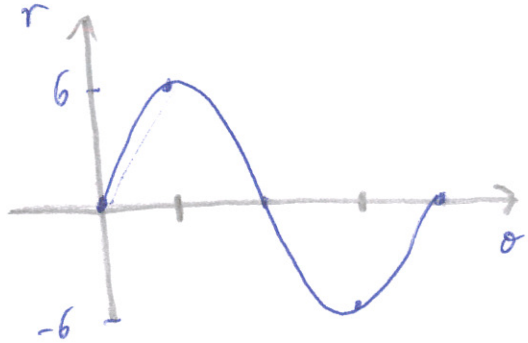
$$= \int_{\pi/6}^{5/6\pi} 8(1-\cos(2\theta))\,d\theta - 2\left(\frac{5}{6}\pi - \frac{\pi}{6}\right) + \left[4\cos\theta\right]_{\pi/6}^{5/6\pi}$$

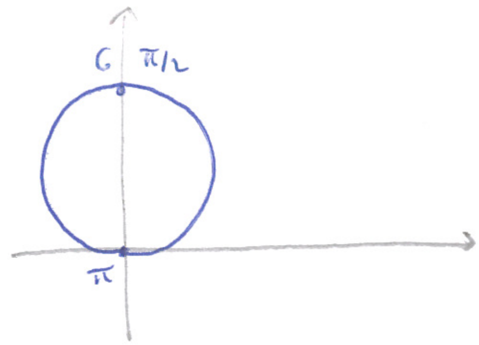
$$= 8\left(\frac{5}{6}\pi - \frac{\pi}{6}\right) - \left[4\sin(2\theta)\right]_{\pi/6}^{5/6\pi} - 2\cdot\frac{2}{3}\pi + 4\cos\left(\frac{5}{6}\pi\right) - 4\cos\left(\frac{\pi}{6}\right)$$

$$= 8\cdot\frac{2}{3}\pi - 2\cdot\frac{2}{3}\pi - 4\sin\left(\frac{5}{3}\pi\right) + 4\sin\left(\pi/3\right) + 4\left(-\frac{\sqrt{3}}{2}\right) - 4\cdot\frac{\sqrt{3}}{2}$$

$$= 6\cdot\frac{2}{3}\pi + \cancel{4\frac{\sqrt{3}}{2}} + \cancel{4\frac{\sqrt{3}}{2}} - \cancel{4\frac{\sqrt{3}}{2}} - \cancel{4\frac{\sqrt{3}}{2}} = 4\pi$$

$r = 6\sin\theta$





$r = 2 + 2\sin\theta$